Midterm Quiz

New Beginnings Theory, Summer 2018

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August 6^{th}, 2018

Name ____

Problem 1 True/False. If true, explain why. If false, give a counterexample.

a. A valid argument can have a false conclusion.

b. An invalid argument cannot have a false conclusion.

c. A pair of equivalent sentences can be derived from each other using the proof rules.

d. The set $A = \{\emptyset\}$ contains one proper subset.

e. The *part of*, e.g. Oregon is a part of the USA, relation is antisymmetric and transitive.

f. Lexicographical order, i.e. dictionary order, is a partial order but not a strict order.

g. *Being the same age as* is an equivalence relation.

Problem 2 Determine whether each of the statements is true. Then demonstrate either with a proof or an example (depending on what you're demonstrating).

a. $A \vee C$, $C \to D$, $D \to A \vdash A$

b. $(\neg A \wedge B) \to C$, $(A \wedge B) \to D$, $(B \wedge D) \to C$, $\vdash B \to C$

c. $\neg(P \to Q)$ is equivalent to $P \wedge \neg Q$

d. The following set of sentences is jointly possible (logically consistent):
$A \vee B$
$A \to C$
$(A \wedge C) \to B$
$B \to (A \wedge \neg C)$

e. $(P \to Q) \vee (Q \to R)$ is necessarily true.

f. $(P \vee Q) \to (P \wedge Q)$ is necessarily true.

Problem 3 Symbolize the following arguments and determine whether valid or invalid.

a. If the earth were spherical, it would cast curved shadows on the moon. It casts curved shadows on the moon. Therefore, the earth must be spherical.

b. If the plasmodium parasite is found in all victims of malaria, but not in other people, then it is the source of the disease. If the plasmodium parasite is found in the anopheles mosquito and it is the source of the disease, then we should eradicate the anopheles. So we should eradicate the anopheles mosquito, if the plasmodium parasite is found in them and all victims of malaria but not in people who do not have malaria.

Problem 4 Suppose A, B, and C represent three bus routes through Portland. Let A, B, and C also be sets whose elements are the bus stops for the corresponding bus route. Suppose A has 25 stops, B has 30 stops, and C has 40 stops. Suppose further that A and B have 6 stops in common, A and C have 5 stops in common, B and C have 4 stops in common, and A, B, and C have 2 stops in common.

a How many distinct stops are there on the three bus routes?

b How many stops for A are not stops for both B and C

Problem 5 Decide which of the following functions are: injective (one-to-one), surjective (onto), both (bijective), or neither.

a $f : \mathbb{Z}^2 \to \mathbb{Z}$ where $f(m, n) = m - n$

b $g : \mathbb{Z}^2 \to \mathbb{Z}^2$ where $g(m, n) = \langle n, m \rangle$

c $h : \mathbb{Z} \times (\mathbb{Z} \setminus 0) \to \mathbb{Q}$ where $h(m, n) = m/n$

d $k : \mathbb{Z} \to \mathbb{Z}^2$ where $k(n) = \langle n, n \rangle$

Problem 6 For each $n \in \mathbb{N}$, let $D_n = \{n, 2n, 3n, ...\}$ = multiples of n

a Find: $D_2 \cap D_7$

b Find: $D_6 \cap D_8$

c Find: $D_3 \cup D_{12}$

Extra Credit Prove the following identity statement is true: $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Extra Credit Show that if A is uncountable and B is a countable subset of A, then the set $A \setminus B$ is uncountable. (Hints: Proof by contradiction is a good idea. Also think about how set operations preserve countability.)